韓國數學敎育學會
2013 국제수학영재교육학술대회 프로시딩
(2013. 8. 9–10) 00–00

The Korean Society of Mathematical Education
Proceedings of the 2013 International Conf. on Math. Edu.
on Creativity & Giftedness (August 9–10, 2013) 00–00

Integrating Real-World Numeracy Applications and Modelling into Vocational Courses

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(Received June 23, 2013; Revised Xxx 00, 2013; Accepted Xxx 00, 2013)

Five levels are identified for incorporating applications and modelling into mathematics courses [Tang, A.; Sui, L. & Wang, X. (2003). Teaching patterns of mathematical application and modelling in high school. In: Qi-Xiao Ye, et al. (Eds.), *Mathematical modelling in education and culture: ICTMA 10* (pp. 233–248). Chichester, UK: Horwood Publishing]:
* Extension
* Special Subject
* Investigation Report
* Paper Discussion
* Mini Scientific Research
These represent a progression from applications set by the teacher, through increasing student involvement in the solution of real world problems, to totally independent project work.
Examples are given of the incorporation of these five levels of application to increase vocational students' mathematical creativity and motivation. Case studies are presented from: engineering, construction, computing, and environmental science.

Keywords: numeracy, vocational education, modelling
MESC Classification: M10
MSC2010 Classification: 97 M10

1. INTRODUCTION

This paper describes practitioner research which is being carried out by tutors of vocational courses at a Further Education College in Wales. Students often begin vocational courses with a poor experience of school mathematics and lack enthusiasm to improve their mathematical skills. However, numeracy and problem solving may be an essential aspect of their vocational training, for example: in subjects such as engineering or construction. The aim of the current project is to develop a framework of learning strategies which will interest and motivate students, develop their numeracy skills within their vocational areas, and help them to gain transferrable skills in critical thinking, creativity, teamwork and collaboration, and learning self-direction.

School mathematics in Britain, as in many other countries, is designed around a bottom-up academic model. Pupils learn mathematical methods within distinct topic areas such as: number, algebra and geometry, then work on example applications still within these same topic areas. The intention is that pupils will progress to study subjects at an advanced level, such as sciences, where they will be able to make good use of the mathematical techniques they have learned.

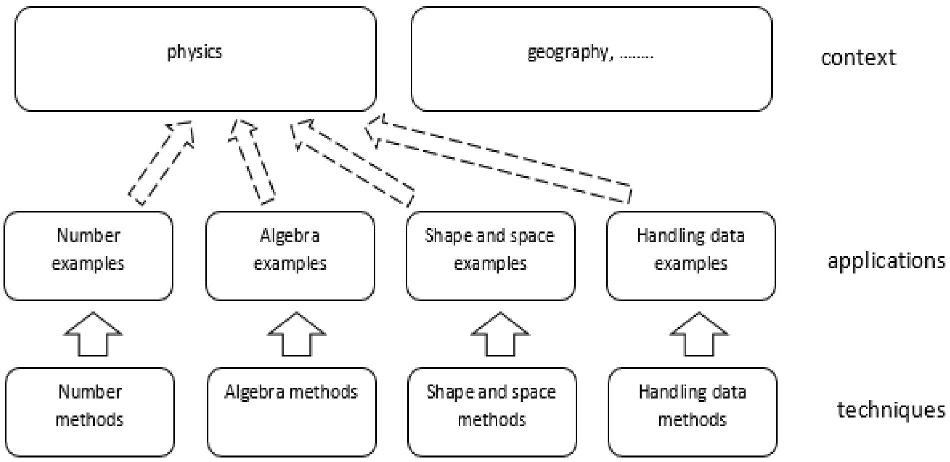

Figure 1. Bottom-up academic mathematics model

This model can present problems for students who leave school at the age of 16 to study a practical vocational course. They may view mathematics as a series of unrelated topics, some of which seem to have no relevance to their chosen profession. Algebra, in particular, is seen by many school leavers as having very little practical everyday use.

Clinical interviews (Ginsburg, 1981) were carried out with 12 students entering a range of vocational courses. The students were asked to give a commentary on their reasoning whilst attempting to solve various mathematical problems. From an analysis of the interview transcripts, four particular difficulties were identified:

- Lack of specialised mathematical vocabulary. Students had difficulty describing features of graphs, equations and other mathematical entities.
- No strong connection between number and algebra in problem solving (Lee & Wheeler, 1989). Students made no attempt to understand relationships in formulae by substituting numerical values, and made no attempt to devise
- Formulae to simplify the repetitive handling of numerical data.

- A preference for justification by concrete example. Students generally preferred to use manipulation and measurement of solid shapes to solve problems, rather than abstract mathematical reasoning.
- Misuse of standard algorithms which had been learned in a superficial manner without full understanding. Examples causing difficulty included formulae for areas and volumes, sides of triangles, and trigonometry.

It became evident that there was little to be gained by continuing to teach in a way which had already been unsuccessful for some students. A new approach was therefore attempted, and forms the basis of this paper.

2. APPLYING NUMERACY

As an alternative to the study of mathematical methods, the approach taken was to work downwards within the vocational area to identify numeracy tasks that practitioners might need to undertake in their everyday work. The tasks were then analysed by the students, and solved using mathematical methods which might be familiar or which might need to be learned at this point. Additionally, the work provided opportunities for consolidating mathematical knowledge in these broader topic areas.

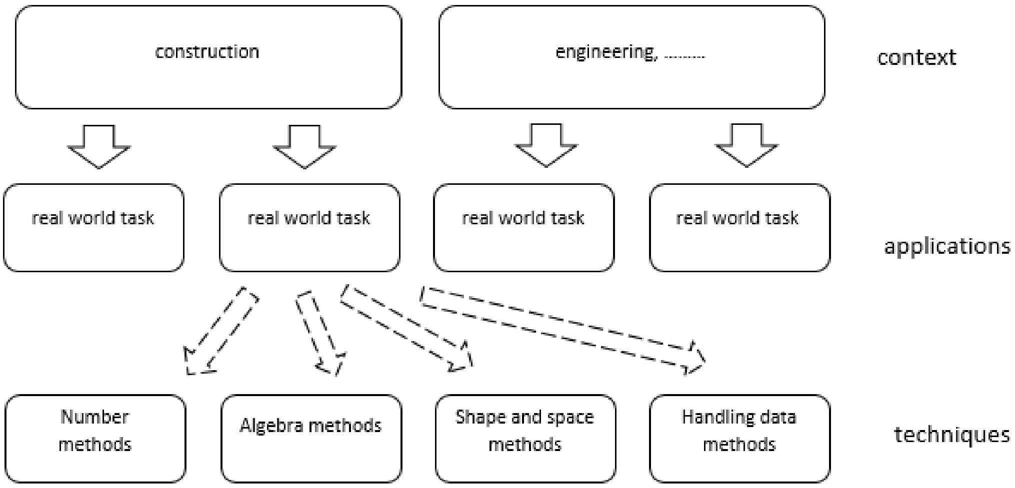

Figure 2. Top-down vocational numeracy model

A distinction is made here between *mathematics*, which is taken to be a set of quantitative methods, and *numeracy* which has wider links with the real world. Numeracy need not be at an elementary level but might include, for example, the advanced mathematics used by engineers or scientists in the course of their work:

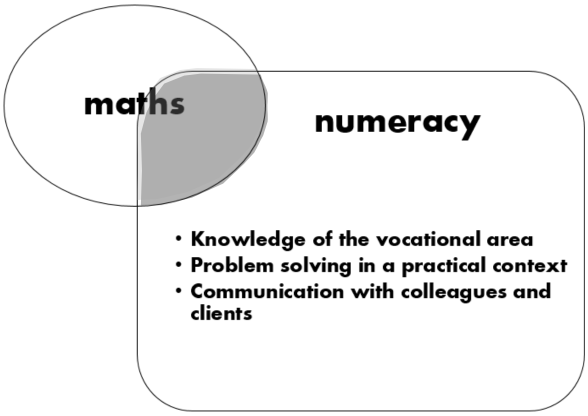

Figure 3. The overlapping relationship between mathematics and numeracy

In addition to applying relevant mathematical methods, numeracy can include the wider skills of practical problem solving, and an ability to communicate with others to determine their requirements, and to present a mathematical solution in a format with is appropriate for effective decision making.

Central to the numeracy approach which we are developing with our students is the MeE motivation model of Martin (2002) and Munns & Martin (2005), summarised in Figure 4:

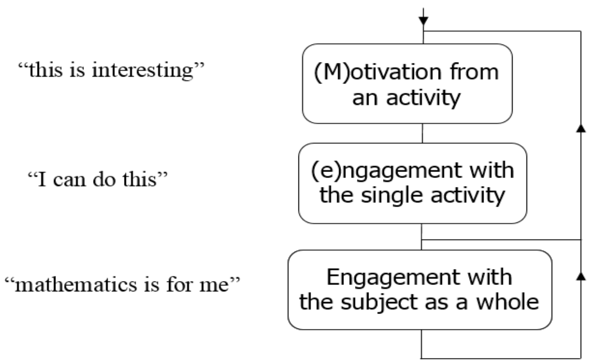

Figure 4. MeE motivation model of Munns & Martin (2005)

The authors advocate the introduction of the most interesting work from the very start of a course, as a means of generating enthusiasm. Tasks may need to be simplified to ensure that students achieve a successful outcome and gain a sense of achievement. It is important that the tasks presented are seen by the students as realistic, relevant and worthwhile.

A number of interesting and motivating tasks might need to be presented, but it is hoped that individual students will reach a point where they engage with the subject as a whole. From this stage on, the work of the teacher becomes much easier. The value of the subject is clear to the students and they become motivated to extend their knowledge and skills through independent learning.

3. NATURALLY OCCURRING NUMERACY

In a number of vocational areas, numeracy tasks occur quite naturally in everyday work. Two examples produced by colleagues are presented here:

Curved work in carpentry (Slaney, 2013)

Amongst the more advanced practical skills taught to carpentry students are methods for constructing curved door and window frames of various designs. Designs have to be produced as a bench template for cutting the timber components.

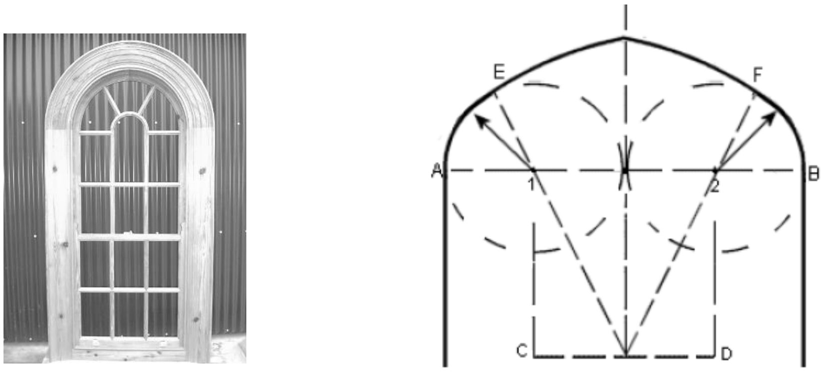

Figure 5. Geometrical construction method for a Tudor arch

Geometrical methods have been developed by stone masons and carpenters since Roman times and earlier for producing various designs of arch. As an example, students can investigate the construction method for a Tudor arch:

- Two circles are drawn to determine the width of the arch,
- A square is constructed, with the distance between the circle centres as one side,
- The centre of the bottom edge CD is used as a centre for constructing the upper arcs to complete the arch.

Students can continue by investigating the geometry of other arch designs as a project.

Expedition planning

Students who are training to become outdoor pursuits instructors are required to make reasonably accurate estimates of the time which expeditions will take over mountainous terrain as part of the procedure for safety planning. A mathematical formula known as *Naismith's Rule* can be used for estimating journey time. This determines a time based on walking speed over flat ground, then adds extra time for the amount of ascent and descent necessary during the journey.

Students using Naismith's Rule have found that the time calculations for expeditions in the mountainous area of North Wales are very inaccurate. This is due to wide variations in the time taken to cross different types of terrain. Walking speed is much slower across moorland, overgrown forest or wetland than along well constructed footpaths. Scrambling over rocks on mountainsides is particularly slow.

As a project, students have documented the actual times taken for the different stages of a number of expedition routes, and have related these to the nature of the terrain. They are attempting to develop a more accurate journey time formula to improve on Naismith's Rule. This project is a good example of the application of the modelling cycle of Blum and Leiß (Keune & Henning, 2003).

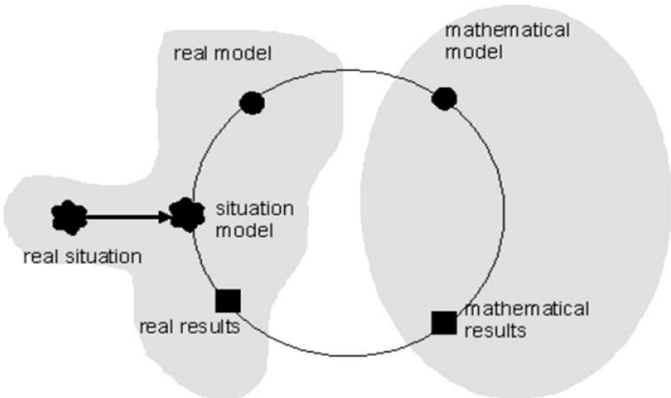

Figure 6. The modelling cycle of Blum and Leiß

The modelling cycle begins in the real world domain, where it is necessary to identify the factors which are important to the outcome of the model. The relationships between these factors are then assessed in descriptive terms. Modelling then moves to the mathematical domain, where the factors are expressed in terms of a formula and numerical examples are run to generate modelling predictions. These modelling predictions are then related back to the real world domain and checked against actual observations. If necessary, the modelling assumptions can be revised and the model re-run, until an acceptable solution is found.

A particular value of this project has been to help students make a connection between algebra and number, with these mathematical methods employed together effectively in problem solving.

4. FRAMEWORK FOR NUMERACY APPLICATIONS BASED ON THE WORK OF TANG, SUI & WANG (2003)

Whilst naturally occurring numeracy tasks are valuable, it is sometimes necessary for tutors to develop additional applications to broaden the mathematical and problem solving skills of vocational students.

Tang, Sui & Wang (2003) proposed a framework which we have adapted for use in vocational courses during this project. The original framework was intended to provide opportunities for mathematics students to apply their quantitative skills in realistic real world situations. Our approach is slightly different, in that we have used the framework as an opportunity for vocational students to experience realistic numeracy problems, and in the process to further develop mathematical skills.

Five levels were identified for incorporating applications and modelling into mathematics courses. These represent a progression from applications set by the teacher, through increasing student involvement in the solution of real world problems, to totally independent project work. Examples will be given from each of the levels:

Extension

In this approach, students who have been studying a mathematical topic are presented with an ill-defined real world problem where they need to seek out additional data for its solution. As an example, consider the following question which might be given at the end of a study of trigonometry:

A photographer is intending to travel on the London Eye, a large Ferris wheel in London. She wishes to take panoramic views across the city, but needs to be at least higher than the roof of the nearby County Hall building to do this. She would like to know how many minutes will be available for the photographic session.

In this case, the student should obtain actual data, or at least reasonable estimates, for the speed of rotation of the wheel, its diameter, and the height of the adjacent building. This might be found by use of the Internet. The student is then free to devise their own method for numerical, graphical or analytic solution of the problem.

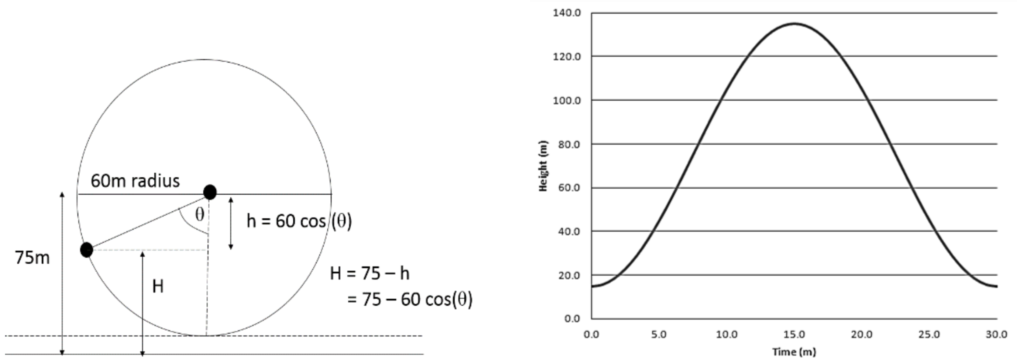

Figure 7. A possible graphical solution of the London Eye problem.
The wheel has a diameter of 120m, and rotates in 30 minutes.

Special Subject

Students who have studied a vocational topic are given the opportunity to investigate the topic further through a quantitative project. This approach was used successfully with construction students who had been studying heat losses from buildings. After discussion of the insulating properties of different building components, students developed their own spreadsheets to determine the heat losses from a house. This allowed investigation of the effects of double glazing of windows, cavity insulation of walls, and insulation of the roof space, and gave a deeper understanding of the mathematics involved.

Figure 8. House heat loss

Investigation Report

For this approach, students gather their own primary data through surveys, laboratory or fieldwork measurements, then process the data using appropriate mathematical methods. In this way, it is hoped to gain a clearer interpretation of the data and to obtain insights which were not initially obvious.

As an example, geography students investigating coastal processes measured pebbles which were being transported along a shingle spit by wave action. It was seen that although a mixture of pebble sizes were present at each location visited, there was a reduction in mean size during transport along the shingle spit. A hypothesis was developed that the rate of size reduction would be proportional to the actual pebble size — large pebbles would be eroded more easily by wave action than small pebbles.

A spreadsheet numerical model was developed for a constant percentage reduction in size for each distance unit, leading to the familiar negative exponential curve. The curve produced in the spreadsheet model was seen to closely reflect the best fit curve through the actual field data, supporting the initial hypothesis connecting rate of erosion directly to pebble size:

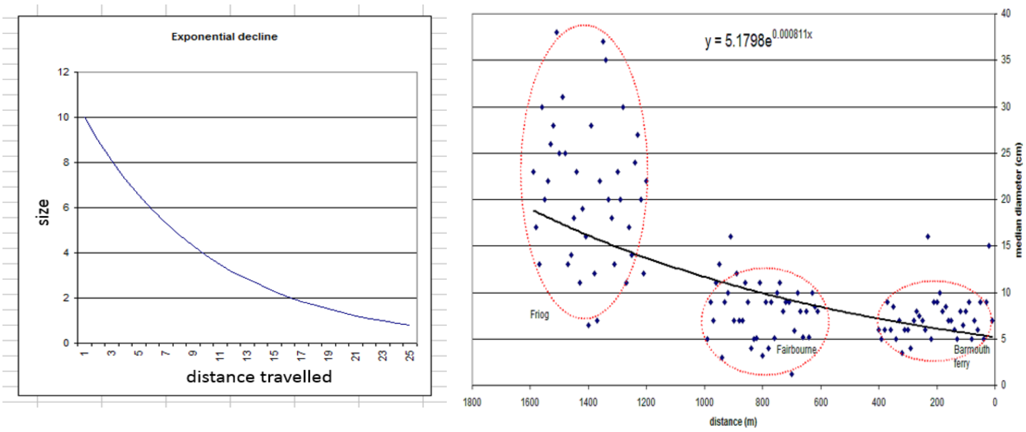

Figure 9. A comparison of a theoretical model and field data for pebble erosion

Paper Discussion

The approach used here is to present students with an interesting and challenging vocational mathematics task, then provide resources from books, journal articles or the Internet which will allow the students to teach themselves the necessary quantitative techniques for solving the problem. This contrasts with the normal teaching approach in which the tutor provides instruction, and is intended to encourage students to develop as independent learners.

An example presented to computing students was to model an epidemic of a non-fatal illness such as influenza. Published articles were provided which explained the recurrence relations which form the Simple Epidemic Model (Keeling, 2001). The population is modelled as three groups:

- Susceptible: those who can catch the illness,
- Infected: those who have the illness, and could infect others, and
- Recovered: those who cannot catch the illness again and are no longer infectious to others.

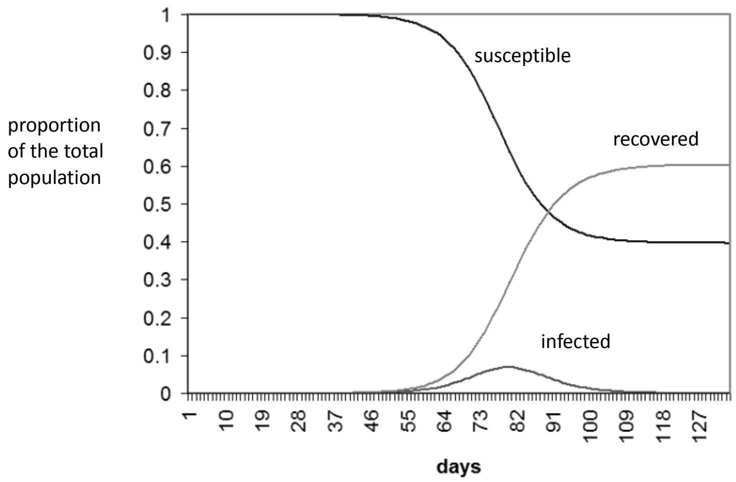

Figure 10. Results generated by students in running the Simple Epidemic model. It is seen that the epidemic begins to decline when the number of recovered people in the population exceeds the number susceptible. This is due to the reducing liklihood of an infected person coming into contact with a susceptible person to spread the illness.

Mini Scientific Research

This approach represents the maximum level of student involvement in the planning, investigation and analysis of data for a substantial numeracy project related to their vocational area.

An example project carried out by engineering students has been the investigation of the motion of a car when passing over a speed hump, in response to the springs and shock absorbers of the car suspension system. These components have different force responses related to extent of spring compression and the velocity of compression of the shock absorber. Students were able to compare the results of spreadsheet modelling with video film which they produced of the actual motion of cars passing over speed humps at different speeds.

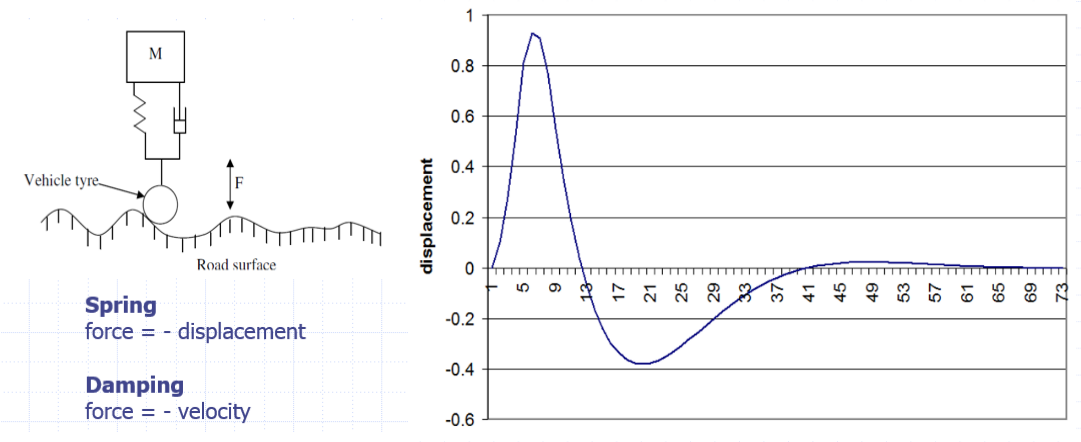

Figure 11. Damped simple harmonic model for the motion of a car passing over a speed hump

5. CONCLUSION

Observations of students undertaking the numeracy tasks were carried out, combined with examination of the solutions produced, and the participants were interviewed about their experiences during the project and their broader attitudes towards numeracy and mathematics. The project is ongoing, but it is clear that high levels of interest and motivation have been generated by the various tasks, and students' confidence in using mathematical techniques has been improved. In particular, problems identified early in the year have been addressed to a significant extent:

- Use of specialised mathematical vocabulary is more evident,
- Numerical and algebraic methods were being combined in the solution of problems,
- Skills in abstract reasoning have improved, and
- A deeper level of understanding of the mathematics used in problem solving is evident.

A difficulty which has not yet been fully resolved is the reconciliation of a problem solving and project based approach to numeracy, and the requirement by some Examination Board syllabuses to assess specific mathematical methods. The creative problem solving approach can still be used as a means of motivating students and developing problem solving skills, but at the end of each project session it may be necessary to allocate time to broader coverage of related mathematical topics. For example, after solving a graphical problem involving an exponential function, the students may be introduced to other related functions such as powers, inverse powers and logarithms. After the use of trigonometry to solve a circular motion problem, students might examine other trigonometric applications in areas such as topographic surveying.

Overall, the development of numeracy through problem solving in vocational areas, either through naturally occurring applications or through the use of the framework of Tang et al. (2003) is seen as an effective way of increasing student motivation and creativity. Further exploration of teaching methods for embedding numeracy in vocational courses is ongoing.

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